Depinning transition of a driven interface in the random-field Ising model around the upper critical dimension

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We investigate the depinning transition for driven interfaces in the random-field Ising model for various dimensions. We consider the order parameter as a function of the control parameter (driving field) and examine the effect of thermal fluctuations. Although thermal fluctuations drive the system away from criticality, the order parameter obeys a certain scaling law for sufficiently low temperatures and the corresponding exponents are determined. Our results suggest that the so-called upper critical dimension of the depinning transition is five and that the systems belongs to the universality class of the quenched Edward-Wilkinson equation.

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I. INTRODUCTION

Driven interfaces in quenched disordered systems display with increasing driving force a transition from a pinned interface to a moving interface (see, e.g., [1] and references therein). This so-called depinning transition is caused by a competition of the driving force and the quenched disorder. The first one tends to move the interface whereas the latter one hinders the movement. Depinning transitions are observed in a large variety of physical problems, such as fluid invasion in porous materials [2], depinning of charge density waves [3,4], impurity pinning of flux line in type-II superconductors [5], contact lines [6] as well as in field driven ferromagnets, where the interface separates regions of opposite magnetizations [7].

A well established model to investigate the depinning transition in disordered ferromagnets is the driven random-field Ising model (RFIM) (see, for instance, Refs. [2,7–10]). Here, the disorder induces some effective energy barriers which suppress the interface motion. A magnetic driving field H reduces these energy barriers but they vanish only if the driving field exceeds the critical value H_c. The transition from the pinned to the moving interface can be described as a continuous phase transition and its velocity v is interpreted as the order parameter. Without thermal fluctuations ($T=0$) the field dependence of the velocity obeys the power-law behavior

$$v(h,T=0)\sim h^{\beta} \tag{1}$$

for $h>0$, where h denotes the reduced driving field $h=H/H_c-1$.

The depinning transition is destroyed in the presence of thermal fluctuations ($T>0$) which may provide the energy needed to overcome local energy barriers. Although thermal fluctuations drive the system away from criticality the order parameter obeys certain scaling laws and for sufficiently low temperatures the order parameter can be described as a generalized homogenous function [11]

$$v(h,T)=\lambda\tilde{v}(\lambda^{-1/\beta}h,\lambda^{-\psi}T), \tag{2}$$

similar to usual equilibrium second-order phase transitions. Setting $\lambda^{-1/\beta}=1$ one recovers Eq. (1) for zero temperature. Choosing $\lambda^{-\psi}T=1$ one gets for the interface velocity at the critical field H_c

$$v(h=0,T)\sim T^{1/\psi}. \tag{3}$$

This power-law behavior was observed in two- and three-dimensional simulations of the driven RFIM [11,12] as well as in charge density waves in computer simulations and mean-field calculations [4,13].

Furthermore, thermal fluctuations cause a creep motion of the interface for small driving fields ($H\ll H_c$) characterized by an Arrhenius-like behavior of the velocity. This creep motion was observed in experiments considering magnetic domain wall motion in thin films composed of Co and Pt layers [14], in renormalization group calculations [15,16] regarding the Edwards-Wilkinson equation with quenched disorder, as well as in numerical simulations of the RFIM [17].

In equilibrium physics a scaling ansatz according to Eq. (2) usually describes the order parameter as a function of its control parameter and of its conjugated field. Although Eq. (2) can be applied to the depinning transition, i.e., the temperature is a relevant scaling field, T is not conjugated to the order parameter. The conjugated field would support the interface motion independent of its strength. But strong thermal fluctuations destroy the interface instead to support the interface motion. Therefore, one has to interpret the value of the thermal exponent ψ carefully. For instance, it is not clear whether the obtained values of ψ are a characteristic feature of the whole universality class of the depinning transition or just a characteristic feature of the particularly considered RFIM. This point could be important for the interpretation of experiments that naturally take place at finite temperatures.

In this paper we reinvestigate the interface dynamics of the driven RFIM and focus our attention to higher dimensions $d\geqslant 3$. In particular, we consider the scaling behavior at the critical point and determine the exponents β and ψ. Our

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results suggest that the so-called upper critical dimension of the depinning transition of the driven RFIM is $d_c=5$. Above this dimension the scaling behavior is characterized by the mean-field exponents. We compare our results with those obtained from a renormalization group approach of the quenched Edward-Wilkinson (QEW) equation that is expected to be in the same universality class as the driven RFIM. A summary is given at the end.

II. MODEL AND SIMULATIONS

We consider the depinning transition RFIM on cubic lattices of linear size L in higher dimensions ($d \geqslant 3$). The Hamiltonian of the RFIM is given by

$$\mathcal{H}=-\frac{J}{2}\sum_{\langle i,j\rangle} S_i S_j - H\sum_i S_i - \sum_i h_i S_i , \tag{4}$$

where the first term characterizes the exchange interaction of neighboring spins ($S_i=\pm 1$). The sum is taken over all pairs of neighbored spins. The spins are coupled to a homogenous driving field H as well as to a quenched random field h_i with $\langle h_i\rangle=0$ and $\langle h_i h_j\rangle \propto \delta_{ij}$. The random field is assumed to be uniformly distributed, i.e., the probability p that the random field at site i takes some value h_i is given by

$$p(h_i)=\begin{cases}(2\Delta)^{-1} & \text{for} \quad |h_i|<\Delta \\ 0 & \text{otherwise.}\end{cases} \tag{5}$$

Using antiperiodic boundary conditions an interface is induced into the system which can be driven by the field H (see, Ref. [11] for details). A Glauber dynamics with random sequential update and heat-bath transition probabilities is applied to simulate the interface motion (see, for instance, Ref. [18]).

A moving interface corresponds to a magnetization M that increases in time t (given in Monte Carlo step per spin). The interface velocity, which is the basic quantity in our investigations, is obtained from the time dependence of the magnetization $v=\langle dM/dt\rangle$ where $\langle\cdots\rangle$ denotes an appropriate disorder average. Starting with a flat interface we performed a sufficient number of updates until the system reaches after a transient regime the steady state which is characterized by a constant average interface velocity.

As pointed out in previous works [11,12] an appropriate choice of the interface orientation is needed in order to recover that the interface moves for arbitrarily small driving field in the absence of disorder. An appropriate choice is to consider the interface motion along the diagonal direction of a simple cubic lattice. For $d \geqslant 3$ an alternative is to examine the interface motion along the z axis on a body-centered cubic (bcc) lattice. Since it is much more convenient to implement the latter case in higher dimensions we consider in this work bcc lattices, the more as the lattice structure usually does not affect the universal scaling behavior.

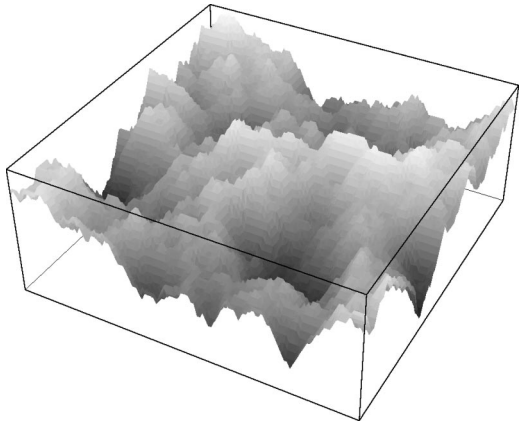

FIG. 1. Snapshot of a moving interface at $T=0$ for $L=128$, $\Delta=1.7$, and $H=1.37$ in $d=3$. In order to show the details we stretched the interface in the vertical direction by a factor 15.

III. $D=3$

In the case of the three-dimensional RFIM, we consider bcc lattices of linear size $L \leqslant 250$. A snapshot of a moving interface in the steady state is presented in Fig. 1. The obtained values of the interface velocities for $T=0$ are plotted in Fig. 2. As one can see v tends to zero in the vicinity of $H \approx 1.36$. Assuming that the scaling behavior of the interface motion is given by Eq. (1) one varies H_c until one gets a straight line in a log-log plot. Convincing results are obtained for $H_c=1.357\pm 0.001$ and the corresponding curve is shown in the inset of Fig. 2. For lower and greater values of H_c we observe significant curvatures in the log-log plot (not shown). In this way we estimate the error bars in the determination of the critical field. A regression analysis yields the value of the order parameter exponent $\beta=0.653\pm 0.026$. This value agrees with $\beta=0.66\pm 0.04$ that was obtained from a similar investigation [11] where the interface moves

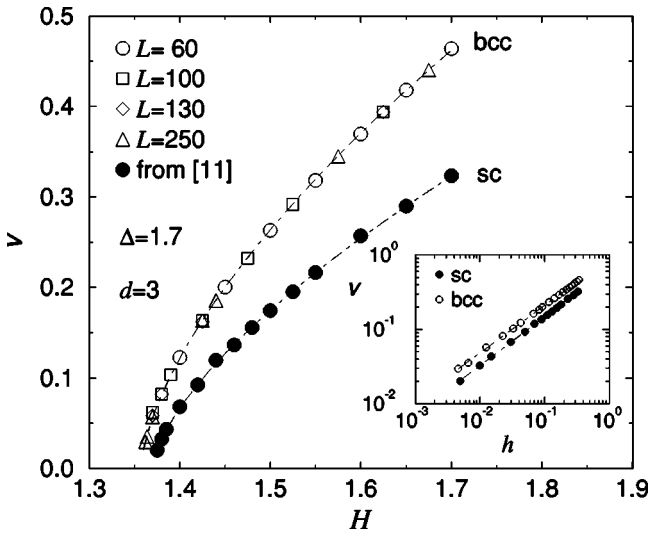

FIG. 2. Dependence of the interface velocity v on the driving field H for a bcc and simple cubic (sc) lattice, respectively. The inset shows v as a function of the reduced driving field h.The dash dotted lines are fits according to Eq. (1).

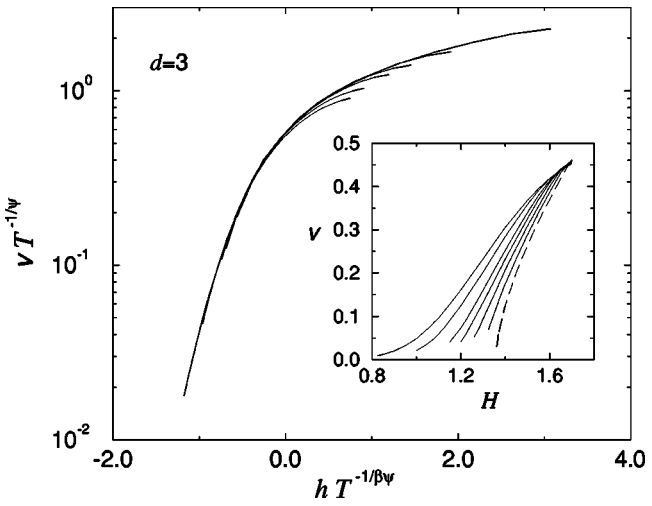

FIG. 3. Scaling plot of the interface velocity for $d=3$. The data are rescaled according to Eq. (6). The inset shows the unscaled velocities for $T=0.025\,n$ with $n\in\{1,2,3,4,6,8\}$ (solid lines) in comparision to the $T=0$ data from Fig. 2 (dashed line).

along the diagonal direction of a simple cubic lattice (see inset of Fig. 2). Furthermore, both results are in agreement with $\beta=0.60\pm0.11$ [9,10], where in $d=3$ the influence of helicoidal boundary conditions in one direction and periodic ones in the other direction parallel to the interface was investigated on a simple cubic lattice.

In order to determine the exponent ψ we simulated the RFIM around the critical field for $T=0.025\,n$ with $n\in\{1,2,3,4,6,8\}$. The obtained curves are shown in the inset of Fig. 3. According to Eq. (2) the interface velocity scales as

$$v(h,T)=T^{1/\psi}\tilde{v}(hT^{-1/\beta\psi},1). \tag{6}$$

Plotting $v(h,T)T^{-1/\psi}$ as a function of $hT^{-1/\beta\psi}$ one varies β, ψ, and H_c until one gets a data collapse of the different curves. Convincing data collapses are observed for $\beta=0.63\pm0.06$ $\psi=2.33\pm0.2$, and $H_C=1.360\pm0.01$ and the corresponding curves are shown in Fig. 3. The obtained values of the order parameter exponent and of the critical field agree within the error bars with the values of the $T=0$ analysis. Furthermore, our results are in agreement with similar investigations on a simple cubic lattice ($\beta=0.63\pm0.07$ and $\psi=2.38\pm0.2$, see Ref. [11]).

IV. $D=4$

In order to determine the order parameter exponent of the four-dimensional driven RFIM we measured the interface velocity for bcc lattices of linear sizes $L\leqslant140$. The obtained data for $T=0$ are shown in a log-log plot in Fig. 4. After a transient regime that displays a finite curvature we observe an asymptotic power-law behavior for sufficiently small driving field H. A regression analysis yields $\beta=0.8\pm0.06$ and $H_c=1.258\pm0.002$.

To determine the exponent ψ we simulated the RFIM in the vicinity of the critical field for $T=0.025\,n$ where again $n\in\{1,2,3,4,6,8\}$ was choosen. Similar to the three-dimensional case one varies the exponents as well as the critical field until one observes a data collapse. Good results are obtained for $\beta=0.73\pm0.13$, $\psi=1.72\pm0.27$, and $H_c=1.256\pm0.015$. The corresponding data collapse is shown in Fig. 5. Again, the obtained values of β and H_c confirm the above presented analysis for $T=0$.

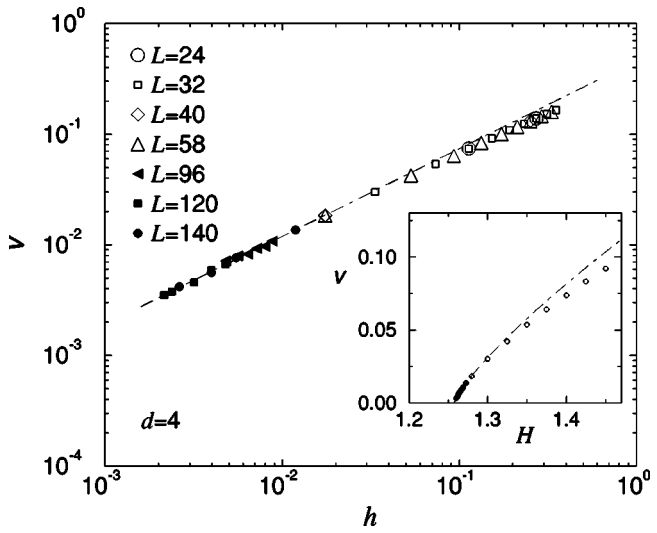

FIG. 4. The interface velocity of the four-dimensional model at $T=0$. For sufficiently small fields the data obey a power law according to Eq. (1) (dotted dashed line). For the fit we use only those data marked by filled symbols and we find $H_c=1.258\pm0.05$ and $\beta=0.8\pm0.06$.

V. $D=5$

In the case of the five-dimensional RFIM system sizes from $L=10$ up to $L=30$ are simulated. Analyzing the interface motion at $T=0$ we observe that the velocity-field dependence cannot be described by a pure power law, i.e.,

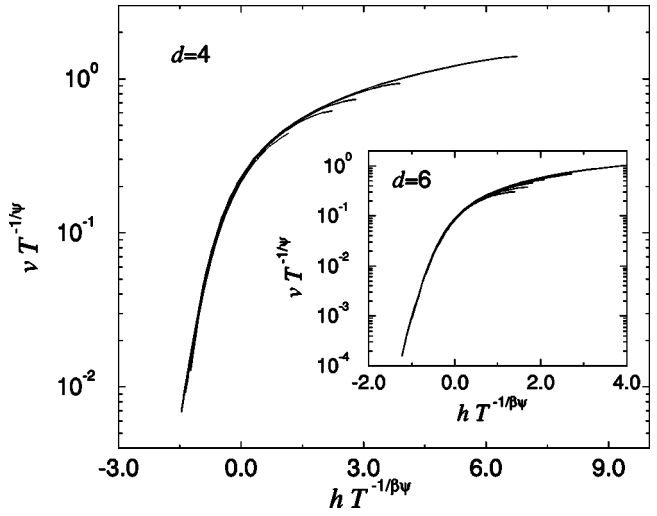

FIG. 5. Scaling plot of the interface velocity for $d=4$ and $d=6$, respectively (see inset). The data are rescaled according to Eq. (6).

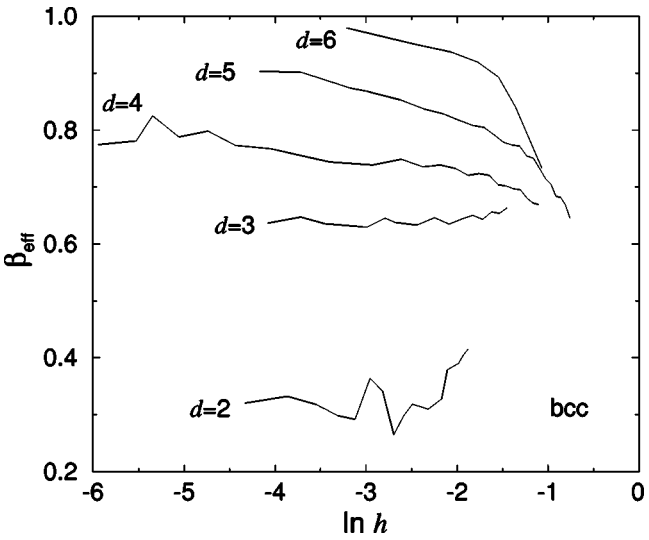

FIG. 6. The effective exponents β_{eff} as function of $\ln h$ for various dimensions. The figure shows that the five-dimensional exponent does not display a saturation as the exponent of the lower dimensions do. The data of the six-dimensional model indicate that the expected saturation value $\beta_{\mathrm{eff}}=1$ is reached significantly faster as compared to $d=5$ (see text). The values of the two-dimensional system are obtained from Ref. [12].

Eq. (1) fails. In Fig. 6 we plotted the logarithmic derivation of the velocity-field dependence,

$$\beta_{\mathrm{eff}}=\frac{\partial \ln v}{\partial \ln h}, \tag{7}$$

which can be interpreted as an effective exponent. If the asymptotic scaling behavior obeys Eq. (1) the logarithmic derivative tends to the value of β for $H\rightarrow H_{\mathrm{c}}$. This behavior is observed in the case of the three- and four-dimensional system (see Fig. 6). For $d=5$ the logarithmic derivative displays no saturation for $H\rightarrow H_{\mathrm{c}}$, i.e., the scaling behavior of the five-dimensional RFIM cannot be described by a simple power-law behavior.

Significant corrections to the usual scaling behavior [Eq. (1)] occur, for instance, at the upper critical dimensional d_{c} where the scaling behavior is governed by the mean-field exponents modified by logarithmic corrections. The scaling behavior around d_{c} is well understood within the renormalization group theory (see, for instance Refs. [19–21]). For $d>d_{\mathrm{c}}$ the stable fix point of the corresponding renormalization equations is usually a trivial fix point with classical mean-field exponents. This trivial fix point is unstable for $d<d_{\mathrm{c}}$ and a different stable fix point exists with nonclassical exponents. These exponents can be estimated by an ϵ expansion, for instance. For $d=d_{\mathrm{c}}$ both fix points are identical and marginally stable. In this case the asymptotic form of the thermodynamic functions is given by the mean-field (MF) power-law behavior modified by logarithmic corrections, e.g.,

$$v(h,T=0)\sim h^{\beta_{\mathrm{MF}}}|\ln h|^{\mathrm{B}}, \tag{8}$$

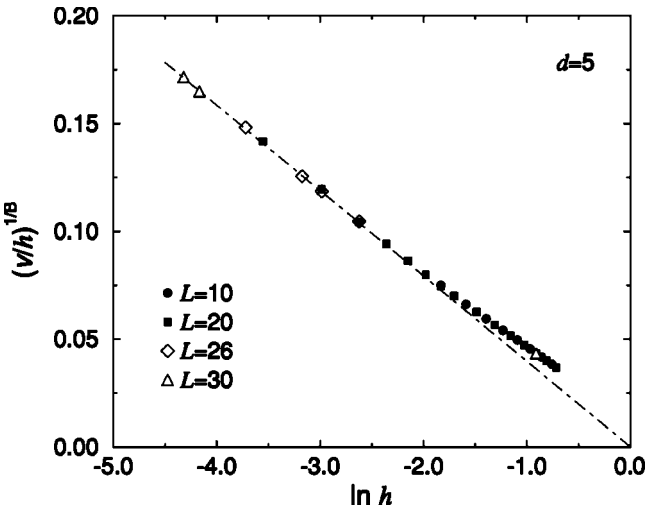

FIG. 7. The rescaled interface velocity as a function of the driving field for $d=5$. In order to display the logarithmic corrections we plot $(v/h)^{1/\mathrm{B}}$ vs $-\ln h$ [see Eq. (8)]. The solid line corresponds to the expected asymptotic scaling behavior for $h\rightarrow 0$ [corresponding to $\ln h\rightarrow(-\infty)$] according to Eq. (9).

where B denotes an unknown correction exponent. It is worth to mention that in contrast to the values of the critical exponents below the upper critical dimension the above scaling behavior [Eq. (8)] does not rely on approximation schemes like ϵ or $1/n$ expansions [22]. Within the renormalization group (RG) theory it is an exact result in the limit $h\rightarrow 0$ (see, e.g. [19,23] and references therein for RG investigations and [24,25] for measurements).

The value $\beta_{\mathrm{MF}}=1$ is reported for depinning transitions [1,26,27]. Thus we analyze $v(h)/h$ as a function of $|\ln h|$ and note again that Eq. (8) describes only the leading order of the scaling behavior, i.e., we expect that the asymptotic behavior of interface velocity obeys

$$[v(h)/h]^{1/\mathrm{B}}=\mathrm{const}\ \ |\ln h|. \tag{9}$$

Therefore, we varied in our analysis the logarithmic correction exponent B and the critical field H_{c} until we get this expected asymptotic behavior. The best results are obtained for $B=0.40\pm 0.09$ and $H_{\mathrm{c}}=1.14235\pm 0.001$ and the corresponding scaling plot is shown in Fig. 7.

Similar to the $T=0$ scaling behavior one has to modify for $T>0$ the scaling ansatz since no data collapse could be obtained by plotting the data according to Eq. (2). Motivated by recently performed investigations of the scaling behavior of an absorbing phase transition around the upper critical dimension [28] we assume that the scaling behavior of the order parameter obeys in leading order

$$v(h,T)=T^{1/\psi_{\mathrm{MF}}}|\ln T|^{\sigma}\tilde{v}(x,1), \tag{10}$$

where the scaling argument x is given in leading order by

$$x=hT^{-1/\beta_{\mathrm{MF}}\psi_{\mathrm{MF}}}|\ln T|^{\tau} \tag{11}$$

with $\beta_{\mathrm{MF}}=1$. In our analysis we use the value $\psi_{\mathrm{MF}}=1.49$ obtained from the analysis of the six-dimensional RFIM (see

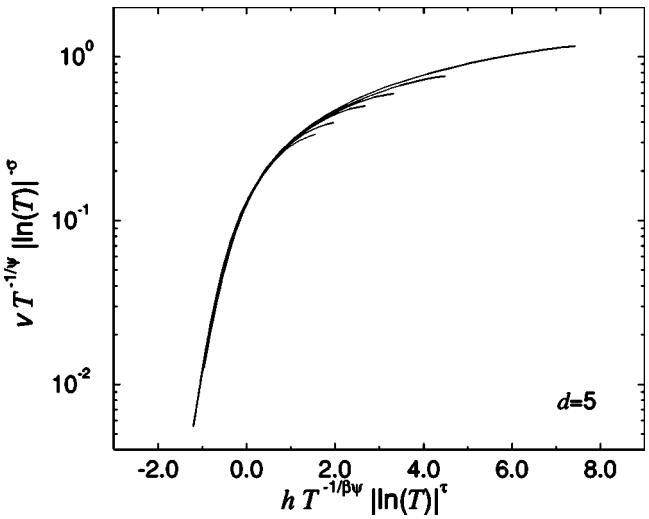

FIG. 8. The scaling plot of the interface velocity v for the five-dimensional model, i.e., at the upper critical dimension. The data are rescaled according to Eqs. (10) and (11) using $\psi=1.49$, $\beta=1$, $\sigma=0.22$, and $\tau=0.19$.

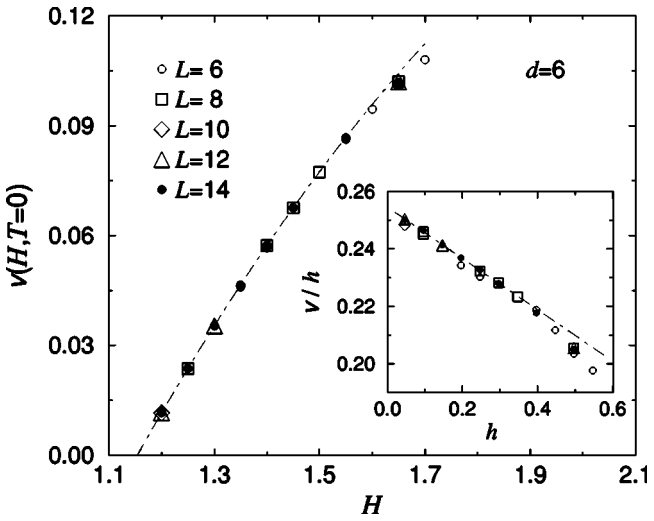

FIG. 9. The interface velocity as a function of the driving field for the six-dimensional model. The dashed line corresponds to a fit according to Eq. (13). The inset displays $v(h)/h$ as a function of the reduced driving field h. The resulting linear behavior confirms that the deviations from the mean-field behavior can be described by quadratic corrections.

the following section). Therefore, we have to vary the exponents σ and τ in order to observe a data collapse according to Eqs. (10) and (11). Convincing results are obtained for $\sigma=0.22\pm0.16$, $\tau=0.19\pm0.12$ The corresponding data collapse is shown in Fig. 8.

VI. $D=6$

Above the upper critical dimension the scaling behavior is characterized by the mean-field exponents, i.e., in leading order the interface velocity is given by

$$v(h,T=0)\sim h. \tag{12}$$

In Fig. 9 we plot the velocity as a function of the driving field H obtained from simulations of system sizes $L\leqslant 14$. As one can see the velocity does not display the expected linear behavior. It seems that a linear behavior is only given for small velocities, i.e., our data do not display the pure asymptotic behavior [Eq. (12)]. This is confirmed by the behavior of the effective exponent [Eq.(7)] that increases fast for $h\to 0$ but the actual saturation to $\beta=1$ does not take place for the considered values of h (see Fig. 6). To observe the asymptotic behavior one has to perform simulations closer to the critical point H_c which requires to simulate larger system sizes.Unfortunately, the limited CPU power makes this impossible.

An alternative is to take the curvature of the function $v(h)$ into consideration and to assume that the leading corrections to the asymptotic behavior are of the form

$$v(h,T=0)=v_1h+v_2h^2+O(h^3), \tag{13}$$

which recovers Eq. (12) for $h\to 0$. Fitting our data to this ansatz we get $H_c=1.1537\pm0.003$, $v_1=0.2937$, and $v_2=-0.1188$. The corresponding curve fits the simulation data quite well as one can see from Fig. 9. In the inset of Fig. 9 we plotted $v(h)/h$ as a function of the reduced driving field h. According to the above ansatz [Eq. (13)] one gets a linear behavior, i.e., the deviations from the pure mean-field behavior [Eq. (12)] can really be described as quadratic corrections. Thus we get that our numerical data are consistent with the assumption that the six-dimensional RFIM depinning transition is characterized by the mean-field exponent $\beta_{\mathrm{MF}}=1$.

Again we consider how thermal fluctuations affect the scaling behavior and analyze interface velocities obtained at different temperatures $T=0.025\,n$ with $n\in\{1,2,3,4,6,8\}$. Similar to the situation below the upper critical dimension we assume that the scaling behavior of the interface velocity is given by Eq. (2) where the exponents are given by mean-field values. A convincing data collapse is obtained for $\psi_{\mathrm{MF}}=1.49\pm0.15$ and $H_C=1.153\pm0.02$ and is plotted in the inset of Fig. 5.

VII. DISCUSSION

A well established realization of interface pinning in a disordered media is the so-called QEW equation of motion that was intensively investigated in the last decade [7,15,16,26,27,29]. It is argued that the QEW equation as well as the driven RFIM are characterized by the same critical exponents, i.e., both models belong to the same universality class [7,9]. Renormalization group analyses of the quenched QEW equation [26,27] predict, in accordance with Ref. [7], $d_c=5$ and allow to estimate the critical exponents using an ϵ expansion. A recently performed two-loop renormalization approach yields [27]

$$\beta_{\mathrm{QEW}}=1-\frac{1}{9}\epsilon-0.040\,123\epsilon^2+O(\epsilon^3) \tag{14}$$

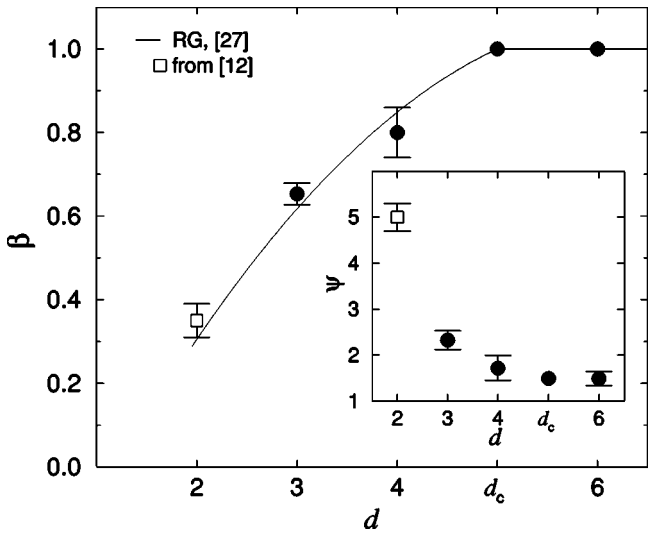

FIG. 10. The critical exponents β and ψ as a function of the dimension. The solid line corresponds to the values of an ϵ expansion [Eq. (14)].

where ϵ denotes the distance from the upper critical dimension, i.e., $\epsilon=5-d$ (unfortunately, no error bars can be estimated from an ϵ expansion). The corresponding values of the exponents as a function of the dimension are plotted in Fig. 10. The numerically determined exponents β of the driven RFIM (listed in Table I) are in a fair agreement with the values of the ϵ expansion.

For the QEW equation the temperature exponent ψ is not known. Therefore, a direct comparison with the obtained values of the driven RFIM is not possible.

VIII. CONCLUSIONS

We studied numerically a field driven interface in the RFIM and determined the order parameter exponent β as well as the temperature exponent ψ. Below the upper critical dimension $d_c=5$ the critical exponents depend on the dimension and the values of the exponents correspond to those of a two-loop renormalization group approach of the Edwards-Wilkinson equation [27]. This suggest that the depinning transition of the RFIM model is in the universality class of the quenched Edward-Wilkinson equation. At the upper critical dimension $d_c=5$ the scaling behavior is affected by logarithmical corrections. Above the upper critical dimension we observe that the scaling behavior is characterized in leading order by the corresponding mean-field exponents.

TABLE I. The critical exponents β (obtained from simulations at $T=0$ and $T>0$, respectively) and ψ of the depinning transition of the RFIM for different dimensions. The values of the two-dimensional model are obtained from Ref. [12]. The critical behavior at the upper critical dimension d_c is additionally affected by logarithmic corrections.

d	$\beta_{T=0}$	$\beta_{T>0}$	ψ
2	0.35 ± 0.04	0.33 ± 0.02	5.00 ± 0.3
3	0.653 ± 0.026	0.63 ± 0.06	2.33 ± 0.2
4	0.80 ± 0.06	0.73 ± 0.13	1.72 ± 0.27
$d_c=5$	1	1	1.49
6	1	1	1.49 ± 0.15

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